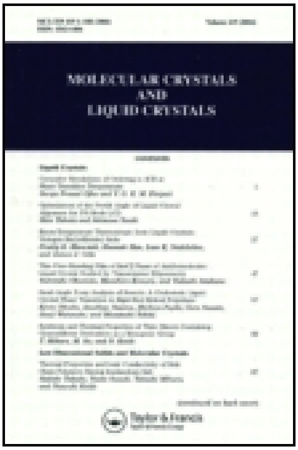

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Small Deformations and Existence of Bistabilities in Liquid Crystal Display Devices

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Small Deformations and Existence of Bistabilities in Liquid Crystal Display Devices

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A model of small deformations is presented which allows the analytical description of some electro-optical properties of liquid crystal display devices (LCD). One of the results is a sufficient condition for the existence of bistabilities in arbitrary LCD's with nonzero surface tilt angles provided strong anchoring. The results are compared with numerical calculations and the importance of device and material parameters is discussed.

INTRODUCTION

In recent literature, there has been an increased interest in the electro-optical properties of liquid crystal display devices based on bistabilities of the director configurations. Many authors have calculated such configurations using routines for solving numerically the nonlinear continuum equations. Based on these calculations, the influence of several device and material parameters was reported.[1,2]

For the case of zero surface tilt, Raynes[3] derived analytical expressions which describe the Fréedericksz transition in supertwist devices. These analytical expressions were used to calculate the combinations of device and material parameters necessary for the existence of bistabilities and for optimum multiplexing performance.[3]

It is the purpose of this paper to set up analytical conditions that are sufficient for the existence of bistabilities using a model for small deformations in LCD's with nonzero surface tilt. The results are compared with the numerical calculations, and the influence of device and material parameters is discussed.

THEORY

We consider a nematic layer of thickness d located between the planes $z = 0$ and $z = d$ of a Cartesian coordinate system. The director is described by the tilt angle θ (measured from the layer plane) and the twist angle Φ. The dielectric constants parallel and perpendicular to the director are denoted by $\epsilon_\parallel$ and $\epsilon_\perp$. The elastic constants for splay, twist and bend are given by k_1, k_2 and k_3. We denote the total twist angle by Φ_T and the pitch of the material induced through a chiral dopant p_0. The tilt angle at both surfaces is denoted by θ_0 and we assume strong anchoring.

Using the abbreviations $q_0 = 2\pi/p_0$, $\rho = (\epsilon_\parallel - \epsilon_\perp)/\epsilon_\perp$ (here supposed to be positive), $\theta_z = d\theta/dz$ and $\Phi_z = d\Phi/dz$ the density of free energy can be written as:

$$f = \{(k_1 \cos^2\theta + k_3 \sin^2\theta)\, \theta_z^2 + \cos^2\theta(k_2 \cos^2\theta + k_3 \sin^2\theta)\, \Phi_z^2 - 2k_2 \cos^2\theta q_0 \Phi_z + k_2 q_0^2 + D_0^2/(\epsilon_0\epsilon_\perp[1 + \rho \sin^2\theta])\}/2 \quad (1)$$

By D_0, we denote the z-component of the dielectric displacement which is constant through the layer. The free energy per unit area is

$$F = \int_0^d f\, dz$$

and we denote by $\bar{f}$ the average value of the energy density $\bar{f} = F/d$. The equations of equilibrium are the conditions of $\bar{f}$ to be stationary.

To study the possible director configurations near the equilibrium state with constant tilt angle $\theta(z) = \theta_0$ through the layer, we use the following ansatz:

$$\begin{aligned} \theta(z) &= \theta_0 + \Delta \sin(\pi z/d) \\ \Phi(z) &= \Phi_T z/d + \sigma \sin(2\pi z/d)/(2\pi) \end{aligned} \quad (2)$$

We assume Δ and σ to be small. It can be seen that $\theta(z)$ and $\Phi(z)$ in Equation 2 satisfy the supposed boundary conditions and represent the first terms of the corresponding Fourier expansions. Δ is the difference between the tilt angle θ_m in the middle of the layer and the boundary tilt angle θ_0. Using Equation 2, we obtain $\bar{f}$ as a function of the parameters Δ and σ. The conditions, necessary to make $\bar{f}$ stationary, are:

$$\left(\frac{\partial \bar{f}}{\partial \Delta}\right)_{\sigma = \text{const}} = 0 \quad \left(\frac{\partial \bar{f}}{\partial \sigma}\right)_{\Delta = \text{const}} = 0 \quad (3)$$

To solve Equation 3 we expand $\bar{f}$ in a series of Δ. For convenience we regard separately two cases.

First case: Nonzero boundary tilt angle ($\theta_0 > 0$)

We use the abbreviations $s_0 = \sin\theta_0$ and $c_0 = \cos\theta_0$ and expand $\bar{f}$ in Equation 3 up to the second order in Δ. We obtain for the equilibrium state:

$$\sigma_E = \frac{8}{3\pi} \Delta \frac{s_0}{c_0} \cdot \frac{\phi_T[(k_3 - 2k_2)c_0^2 - k_3 s_0^2] + k_2 q_0 d}{k_2 c_0^2 + k_3 s_0^2} \quad (4)$$

The voltage U is given by:

$$
\begin{aligned}
\frac{U}{U_H} = 1 + \Delta\Bigg[& \left(\frac{\pi}{2} - \frac{4}{\pi}\right) \frac{\rho s_0 c_0}{1 + \rho s_0^2} \\
& + \frac{\pi}{8 s_0 c_0} \cdot \frac{(k_1 c_0^2 + k_3 s_0^2)\pi^2 - 4(k_3 - k_2)\phi_T^2 s_0^2 c_0^2}{(k_3 - 2k_2)\phi_T^2 c_0^2 - k_3 \phi_T^2 s_0^2 + 2k_2 \phi_T q_0 d} \\
& - \frac{8}{9\pi} \cdot \frac{s_0}{c_0} \frac{\{\phi_r[(k_3 - 2k_2)c_0^2 - k_3 s_0^2] + k_2 q_0 d\}^2}{(k_2 c_0^2 + k_3 s_0^2)\{(k_3 - 2k_2)\phi_T^2 c_0^2 - k_3 \Phi_T^2 s_0^2 + 2k_2 \phi_T q_0 d\}}\Bigg]
\end{aligned}
\tag{5}
$$

Here

$$
U_H = \left[\frac{1}{\epsilon_0(\epsilon_\parallel - \epsilon_\perp)}\{[k_3 - 2k_2)c_0^2 - k_3 s_0^2]\phi_T^2 + 2k_2 \phi_T q_0 d\}\right]^{1/2} \tag{6}
$$

is the voltage for which the tilt angle $\theta(z)$ has the constant value of θ_0 independent of z. The expression for U_H was earlier obtained by Breddels and van Sprang, who studied the corresponding variational equations in the case $\theta(z) = \theta_0$.[4] Denoting by c the capacitance per surface unit area we obtain

$$
\frac{C}{C_H} = 1 + \frac{4}{\pi}\Delta\frac{\rho s_0 c_0}{1 + \rho s_0^2} \qquad C_H = \frac{\epsilon_0 \epsilon_\perp}{d}(1 + \rho s_0^2) \tag{7}
$$

***Second case:* Zero boundary tilt angle ($\theta_0 = 0$)**

In this case we use in Equation 3 the Taylor series of $\bar{f}$ up to the fourth order of Δ. We obtain the same expressions for the Fréedericksz threshold voltage U_{th} as Berreman[5] and for the initial slope of the θ_m *versus* U curve as Raynes[3] who studied the Eulerian equations of the corresponding variational principle.

CONDITIONS FOR THE EXISTENCE OF BISTABILITIES

As we suppose ρ to be positive, the director will turn into the field direction if high voltage is applied, *i.e.*, the difference $\Delta = \theta_m - \theta_0$ increases with increasing voltage U. The existence of a bistability is characterized by a range of voltage in which U becomes smaller with increasing Δ. Therefore it is sufficient that $U/U_{th} < 1$ or $U/U_H < 1$ with $\Delta > 0$. From Equation 5 we obtain the following condition for twisted layers with nonzero boundary tilt:

$$\frac{1}{U_H}\left(\frac{dU}{d\Delta}\right)_{\Delta=0} = \left[\left(\frac{\pi}{2} - \frac{4}{\pi}\right)\frac{\rho s_0 c_0}{1 + \rho s_0^2}\right.$$

$$+ \frac{\pi}{8 s_0 c_0}\frac{(k_1 c_0^2 + k_3 s_0^2)\pi^2 - 4(k_3 - k_2)\phi_T^2 s_0^2 c_0^2}{[(k_3 - 2k_2)c_0^2 - k_3 s_0^2]\phi_T^2 + 2k_2\phi_T q_0 d} \tag{8}$$

$$\left. - \frac{8}{9\pi}\frac{s_0}{c_0}\frac{\{\phi_T[(k_3 - 2k_2)c_0^2 - k_3 s_0^2] + k_2 q_0 d\}^2}{(k_2 c_0^2 + k_3 s_0^2)\{[(k_3 - 2k_2)c_0^2 - k_3 s_0^2]\phi_T^2 + 2k_2\phi_T q_0 d\}}\right] < 0$$

Note that for vanishing boundary tilt, the results of Raynes[3] show that $(dU/d\Delta)_{\Delta=0+0} = 0$. In this case we have $\Delta \equiv 0$ for $U < U_{th}$ and hence $(dU/d\Delta)_{\Delta=0-0} = \infty$. It can be seen in Equation 8 that in the limit, $\theta_0 \rightarrow 0$, the derivative $(dU/d\Delta)_{\Delta=0}$ will be infinite.

This divergence can be explained regarding the relation

$$U_H(\theta_0 = 0) = [U_{th}^2 - k_1\pi^2/(\epsilon_0(\epsilon_\parallel - \epsilon_\perp))]^{1/2} \tag{9}$$

obtained from the expression for U_{th}^5 and Equation 6. This means that $U_H(\theta_0 = 0) < U_{th}$ and hence the limit, $\theta_0 \rightarrow 0$, in Equation 8 describes configurations with an applied voltage below U_{th}. For this configuration we have $\Delta = 0$ and hence we have an infinite derivative.

DISCUSSION

The supertwisted birefringence effect (SBE, see *e.g.*, Waters, Raynes and Brimmel[1] and Scheffer and Nehring[6]) is based on high twisted cells with nonzero boundary tilt angles. To our knowledge, Equation 8 is the first analytical expression for calculating the derivative $(dU/d\Delta)_{\Delta=0}$ and thus the first analytically sufficient condition for the existence of bistabilities in LCD's with nonzero boundary tilt angles. The results obtained by Equation 8 were compared with numerical computations and found to agree excellently.

From Figure 1, it can be seen that there are also parameter sets with bistabilities, although $(dU/d\Delta)_{\Delta=0}$ is positive. In this case, the bistable range is due to higher deformations, while small deformations lead to stable equilibrium states. Small deformations in LCD's with a parameter set fulfilling Equation 8 lead to unstable equilibrium states that are not characterized by a minimum of $\bar{f}$. To obtain large optical contrast in SBE devices, one uses parameter sets fulfilling Equation 8 which result in a relatively large jump in the θ_m *versus* U curve.[2,6]

To illustrate Equation 8, we discuss the dependence on θ_0 of the derivative and the hysteresis width for the following parameter set: $k_1 = 10$pN, $k_2 = 8.7$pN, $k_3 = 21$pN, $\epsilon_\parallel = 20$, $\epsilon_\perp = 10$, $\Phi_T = q_0 d = 3\pi/2$. Using Equation 8 one can see that

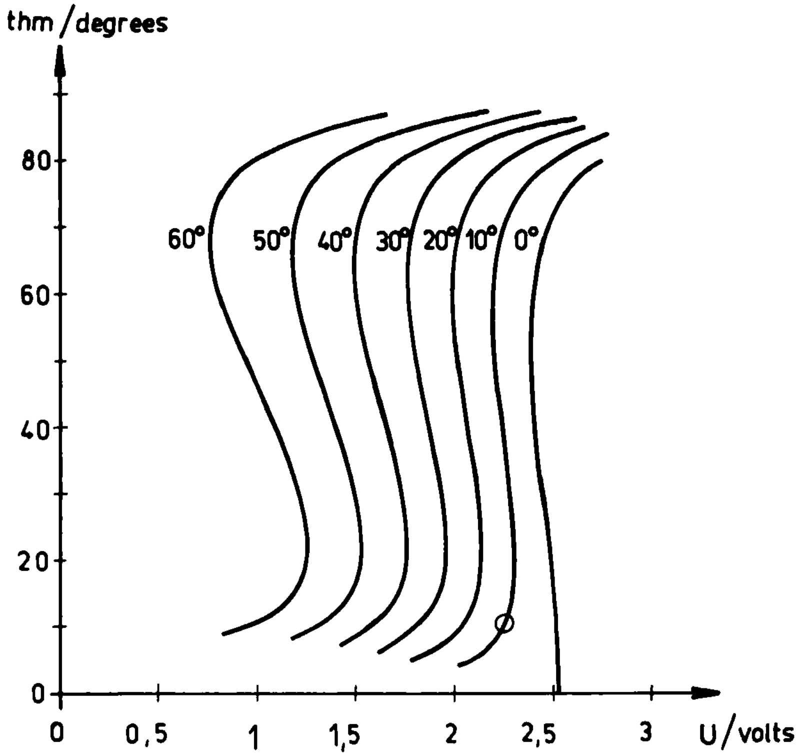

FIGURE 1 Theoretical curves of θ_m *vs* U for k_1 = 10pN, k_2 = 8.7pN, k_3 = 21pN, $\epsilon_\parallel$ = 20, $\epsilon_\perp$ = 10, $\Phi_T = q_0 d = 3\pi/2$ and boundary tilt angles $\theta_0 = 0° \ldots 60°$. (The tilt angle θ_m in the middle of the layer is here denoted by thm.) The circle denotes the homogeneous state $\theta(z) = \theta_m = \theta_0$ for the layer with pretilt angle $\theta_0 = 10°$. Although in this state $dU/d\Delta > 0$, a bistability exists for higher deformations.

for increasing values of θ_0, the derivative decreases. For $\theta_0 > 20°$, this derivative becomes negative, marking a hystersis in the θ_m *versus* U curve. Numerical computations show (Figure 1) that even for low boundary tilt angles there is a hysteresis due to higher deformations. In contrast to van Sprang and Breddels,[2] we have found increasing hysteresis width with increasing θ_0 up to a value of θ_0 for which U_{th} becomes zero. In this case, the bistability already exists for zero voltage and there is no hysteresis in the θ_m *versus* U curve.[7] (Note that for $\theta_0 = 0$, the hysteresis width is larger than for low nonzero boundary tilt angles due to the Fréedericksz transition.)

From Equation 8 it can also be seen that for $k_3 > k_2$ and low dielectric anisotropy, high total twist angles may lead to bistabilities.

Optimum multiplexing occurs for just vanishing hysteresis and therefore the condition is

$$\left(\frac{dU}{d\Delta}\right)_{\Delta=0} = 0 \qquad (10)$$

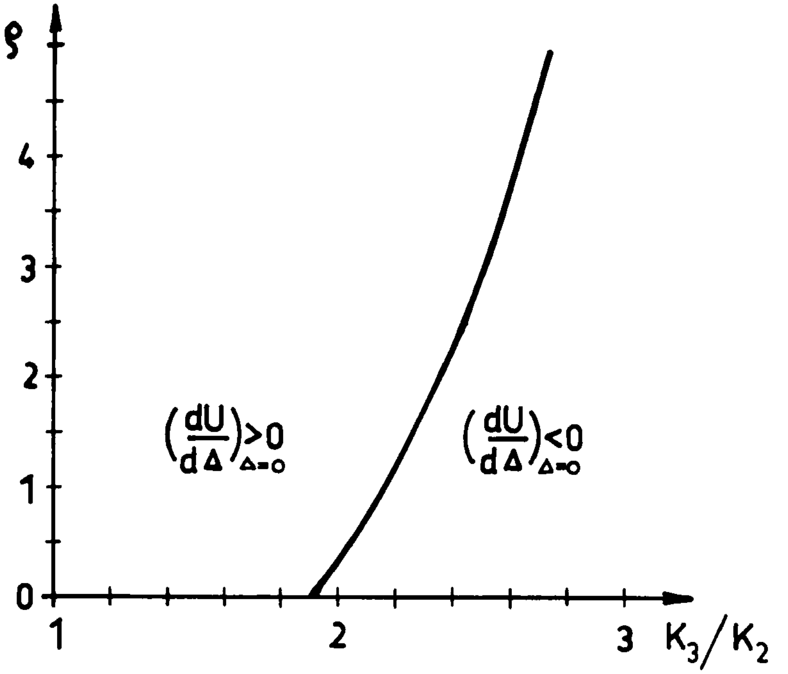

FIGURE 2 Curve $(dU/d\Delta)_{\Delta=0} = 0$ in the $\rho - k_3/k_2$-plane for the following parameter set: $\Phi_T = q_0 d = 270°$, $k_1/k_2 = 1.5$, $\theta_0 = 30°$.

Using Equation 8, one can compute combinations of device and material parameters fulfilling Equation 10. For these parameters, we can expect optimum multiplexing performance. Figure 2 shows the curve with vanishing derivative $(dU/d\Delta)_{\Delta=0}$ in the $\rho - k_3/k_2$ - plane for SBE devices with $\Phi_T = 270°$ and $\theta_p = 30°$, while the other parameters were kept at the following typical values: $k_1/k_2 = 1.5$, and $q_0 d = \Phi_T$. It can be seen that the value of k_3/k_2 necessary for $(dU/d\Delta)_{\Delta=0} = 0$ is between 2 and 3 and is increasing with increasing ρ, similarly to the behaviour of supertwisted layers with zero surface tilt angle.[3]

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